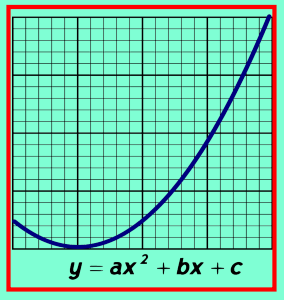
Math 125

Fall 2021

Lecture 36



Class QZ 30

1) Simplify: $3(2-4i) - 2(4+3i)$

$= 6 - 12i - 8 - 6i = \boxed{-2 - 18i}$ ✓

Re. Part $= -2$

Im. Part $= -18$

2) Simplify: $(3-5i)^2 = (3-5i)(3-5i)$

$= 9 - 15i - 15i + 25i^2$

$= 9 - 30i + 25(-1)$

$= \boxed{-16 - 30i}$

Re. $= -16$

Im. $= -30$

3) Solve $x + \sqrt{x+1} = 1$

$\sqrt{x+1} = 1 - x$

Index=2

$(\sqrt{x+1})^2 = (1-x)^2$

$x + 1 = (1-x)(1-x)$

$x + 1 = 1 - x - x + x^2$

$x^2 - 2x + 1 - x - 1 = 0$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ ✓

$x - 3 = 0$

$x = 3$ E.S.

$\{0\}$

Solve & check:

1) $\sqrt{6x+2} - \sqrt{5x+3} = 0$

$\sqrt{6x+2} = \sqrt{5x+3}$

$\left(\sqrt{6x+2}\right)^2 = \left(\sqrt{5x+3}\right)^2$

$6x+2 = 5x+3$

$6x-5x = 3-2$

$\boxed{x=1}$ ✓ $\{1\}$

2) $\sqrt[3]{2x-6} - 4 = 0$

$\sqrt[3]{2x-6} = 4$

Index = 3

$\left(\sqrt[3]{2x-6}\right)^3 = (4)^3$

$2x-6 = 64$

$2x = 70$

$\boxed{x=35}$ ✓ $\{35\}$

Solve and check

$2\sqrt{4x+1} - 9 = x-5$

Always isolate one radical

$2\sqrt{4x+1} = x-5+9$

$\boxed{2\sqrt{4x+1} = x+4}$

$\left(2\sqrt{4x+1}\right)^2 = (x+4)^2$

$4(4x+1) = (x+4)(x+4)$

$16x+4 = x^2+4x+4x+16$

$16x+4 = x^2+8x+16$

$x^2+8x+16-16x-4=0$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$x-6=0$ $\boxed{x=6}$ ✓

$x-2=0$ $\boxed{x=2}$ ✓

$\{2,6\}$

Solve & check:

$2\sqrt{x-3}+4=x+1$

$2\sqrt{x-3}=x+1-4$

$\boxed{2\sqrt{x-3}=x-3}$

→ Square both sides

$\left(2\sqrt{x-3}\right)^2=(x-3)^2$

$4(x-3)=(x-3)(x-3)$

$4x-12=x^2-3x-3x+9$

$x^2-6x+9-4x+12=0$

$x^2-10x+21=0$

$(x-3)(x-7)=0$

$x-3=0$ $\quad x-7=0$

$\boxed{x=3}$ ✓ $\quad \boxed{x=7}$ ✓

$\{3,7\}$

$a+bi$

Divide:

$$\frac{3-2i}{1+3i}=\frac{(3-2i)(1-3i)}{(1+3i)(1-3i)}=\frac{3-9i-2i+6i^2}{1-3i+3i-9i^2}$$

$$=\frac{3-11i+6(-1)}{1-9(-1)}=\frac{3-11i-6}{1+9}=\frac{-3-11i}{10}=\boxed{-\frac{3}{10}-\frac{11}{10}i}$$

Simplify

1) i^{150} even

$=(i^2)^{75}$

$=(-1)^{75}$

$=\boxed{-1}$

2) i^{75} odd

even → $i^{74}\cdot i$

$=(i^2)^{37}\cdot i$

$=(-1)^{37}\cdot i$

$=-1\cdot i=\boxed{-i}$

Rationalize the denominator

1) $\frac{-10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{-10\sqrt{5}}{\sqrt{25}} = \frac{-10\sqrt{5}}{5}$

$= \boxed{-2\sqrt{5}}$

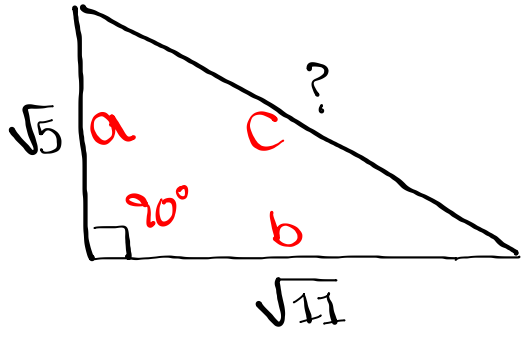
2) $\frac{\sqrt{3}}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{\sqrt{9}-\sqrt{3}}{\sqrt{9}-\sqrt{3}+\sqrt{3}-1}$

$= \frac{3-\sqrt{3}}{3-1} = \boxed{\frac{3-\sqrt{3}}{2}}$

Find the missing Side:



Triangle

Right-Triangle

Pythagorean Thrm

$a^2 + b^2 = c^2$

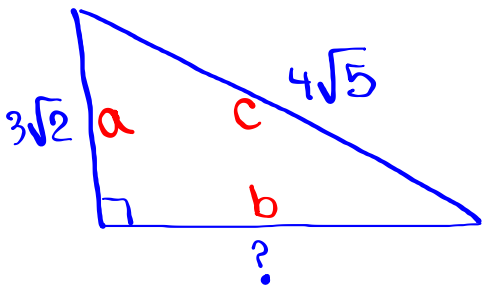
$(\sqrt{5})^2 + (\sqrt{11})^2 = c^2$

$5 + 11 = c^2$

$c^2 = 16$

$\boxed{c = 4}$

Find the missing side:



Right-Triangle Pythagorean Thrm

$a^2 + b^2 = c^2$

$(3\sqrt{2})^2 + b^2 = (4\sqrt{5})^2$

$9 \cdot 2 + b^2 = 16 \cdot 5$

$18 + b^2 = 80$

$b^2 = 80 - 18$

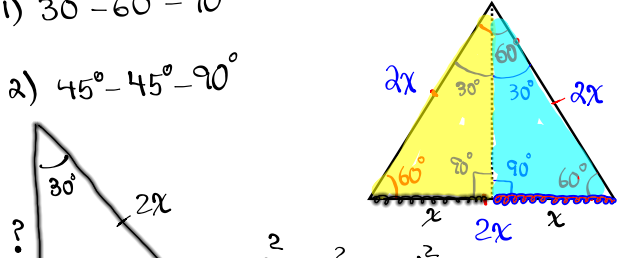
$b^2 = 62$

$\boxed{b = \sqrt{62}}$

Special Right Triangles:

1) $30^\circ - 60^\circ - 90^\circ$

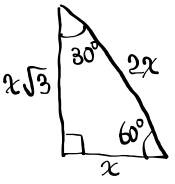
2) $45^\circ - 45^\circ - 90^\circ$



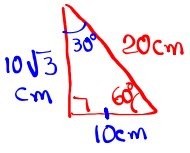
$?^2 + x^2 = (2x)^2$

$?^2 + x^2 = 4x^2$

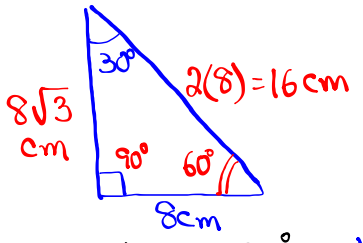
$?^2 = 3x^2$ $\quad ? = x\sqrt{3}$



Side opposite to 30° angle is always half of hypotenuse other side is $\sqrt{3}$ times the other leg.



Find missing angle, and missing Sides.



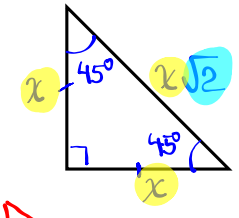
Now 45° – 45° – 90°

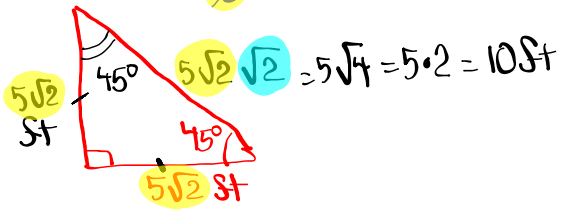
Using Pythagorean Thm

$x^2 + x^2 = ?^2$

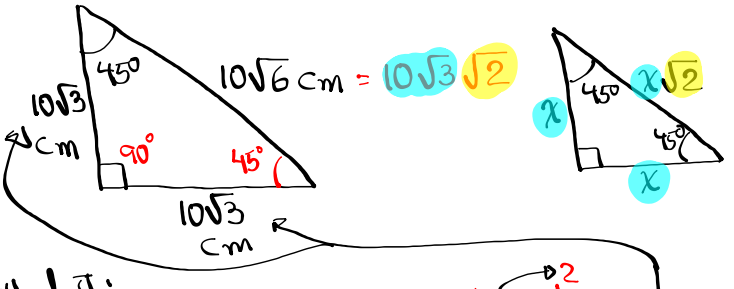
$2x^2 = ?^2$

$? = x\sqrt{2}$

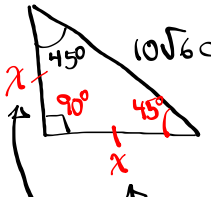




Find missing sides & missing angle:



Method II:



$x^2 + x^2 = (10\sqrt{6})^2$

$2x^2 = 100 \cdot 6$

$x^2 = \frac{100 \cdot 6}{2}$

$x^2 = 300$

$x = \sqrt{300} = \sqrt{100}\sqrt{3}$

$\boxed{x = 10\sqrt{3}}$

Class QZ 31

1) Divide: $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1-i-i+i^2}{1-i+i-i^2} = \frac{-2i}{2} = \boxed{-i}$

2) Simplify: $i^{400} = (i^2)^{200} = (-1)^{200} = \boxed{1}$

3) Simplify: $i^{11} = i^{10} \cdot i = (i^2)^5 \cdot i = (-1)^5 \cdot i = \boxed{-i}$